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Multiple forward scattering of scalar waves through inhomogeneously random burstlike media

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The study of wave propagation through inhomogeneously random media is of importance in actual propagation scenarios, where small- and large-scale inhomogeneities, inadequate sampling, and source and detector fluctuations cannot be avoided. The problem, formulated as propagation through a longitudinally correlated non-Gaussian media, is addressed by using Feynman's path integral, and where applicable, by equivalent partial-differential equations. [S1063-651X(96)05005-4]

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I. INTRODUCTION

This paper presents a comprehensive theory for forward multiple scattering of scalar waves in random media. The scope of propagation through random media covers the planetary and astrophysical observations. The random medium can be either the Earth, or the turbulent atmosphere, or the oceanic internal waves, or the ionospheric electron concentration, or various clouds in the interstellar medium, or the random gravitational lenses.

Whereas the study of lightwave propagation in random media began in 1960, only a few papers presented theories on propagation through inhomogeneously random media before 1995 [1–3]. The inhomogeneous statistics are probably the most undesirable features of a random phenomenon. Including them in the propagation process, instead of avoiding them, is a nontrivial generalization of the theory.

The model of light propagation through large-scale inhomogeneities is described accurately by the narrow-angle approximation of the Helmholtz equation. This approximation, which will be discussed briefly later, leads to the Schrödinger wave equation, where the range is the timelike variable, and the fluctuating refractivity acts like a scattering potential. Therefore our techniques may be useful in the studies of particles that move in inhomogeneously random potentials.

Taking into account nonstationary processes poses a difficulty in modeling, and the interpretation of data. There are two ways to handle the nonstationarity: (a) conditional sampling that intends to validate a limited process that is well defined, or (b) randomization of this process to obtain a composite phenomenon. This paper adopts (b), which gives a wider physical interpretation of actual processes.

When a locally homogeneous scale is identified, we can obtain local averages, i.e., mean, variance, etc. These quantities are slowly modulated by the large-scale variability [3]. The split into small and large scales offers a natural procedure to measure the parameters of the medium: whereas small-scale variabilities are measured with a high resolution but over a small extent, the large-scale variability is measured over a large extent, albeit with cruder resolution. When a locally homogeneous scale is unidentifiable, we need to synthesize a process that will emulate realistic small- and large-scale fluctuations of the medium. This is not a trivial task, because in addition to needing the correct physical characteristics, we must be able to derive the characteristic functional of the medium. Moreover, the large-scale process is not universal, and is influenced by external factors particular to the propagation arena.

Propagation through an inhomogeneously random medium produces longitudinal correlation, which cannot be ignored. It is pronounced when a few large-scale events occur. A multitude of events along the path act to homogenize the propagation, creating an effective medium with a scale of homogeneity that is larger than the homogeneous small-scale length. In this case, the large-scale δ-correlated process is applicable. This model leads to the Markov approximation for the propagation [2], where partial-differential equations can be found in the same way used to derive the Schrödinger equation from Feynman's path integral [5].

In this paper we adopted a plausible model for intermittent refractivity as a superposition of a small-scale Gaussian process and a large-scale filtered point process. The small-scale Gaussian model is customary in studies of propagation through randomly homogeneous media, but the filtered point process for the large-scale variability is an important generalization. It allows us to account for bursts that modulate the small-scale process. We argue that these processes should be correlated to maintain the cascading of energy from large to small scales. This assumption provides us with the information that is necessary to analyze turbulence data in order to parametrize the model.

In this paper, Sec. II provides the basic definitions and equation of wave fields propagating through random media. Section III describes the propagation through inhomogeneously random media, and the limit of δ-correlated process. Section IV introduces the models for small- and large-scale variabilities, and a few examples. We conclude this paper with a short summary.

II. BASIC DEFINITIONS AND EQUATIONS

Consider a wave $\psi(z,x)$ impinging on a turbulent half space $z>Z'$. Forward scattering under the narrow-angle approximation of the Helmholtz equation can be described by

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the two-dimensional Schrödinger wave equation (e.g., Ref. [2]),

$$\left[i\partial_{\mathbf{z}} + \frac{1}{2k}\,\Delta_{\mathbf{x}} + \frac{k}{2}\,\chi(z,x;\mathbf{a}) \right]\psi = 0, \tag{1a}$$

with the initial condition

$$\psi(z=Z',\mathbf{x}) = \psi_0(x,\mathbf{a}_0) \in L_2(\mathbb{R}^2). \tag{1b}$$

The validity of (1) is well established; the narrow-angle approximation is valid when $l \gg \lambda$, where l is the smallest scale of the medium, and λ is the wavelength. Under this approximation, the backscattering and polarization effects are negligible.

Equations (1a) and (1b) comprise a Cauchy problem. The timelike variable $z \in [Z',Z'']$ denotes the range. The spacelike variable $x \in \mathbb{R}^2$ is a transverse position vector. The parametric variability $\chi(z,x;\mathbf{a})$ is the random lossless electric susceptibility with a formal random variable $\mathbf{a}$ defined over an appropriate probability space. In (1a), k is a reference wave number, and Δ_x is a two-dimensional Laplacian. In (1b), the formal random parameter $\mathbf{a}_0$ is assumed to be independent of $\mathbf{a}$. Although in a different context, this equation is similar to the time-dependent Schrödinger equation with a random potential.

A. The statistical moments

Because the solutions of (1) are stochastic, measurable quantities are sought in terms of the statistical moments. The product of instantaneous fields at the same propagation plane, z, is defined by

$$\gamma(z,\mathbf{x},\mathbf{y};\mathbf{A}) \equiv \prod_{n=1}^{N} \psi(z,x_n;\mathbf{A}) \prod_{m=1}^{M} \psi^*(\hat{z},y_m;\mathbf{A}). \tag{2}$$

In (2), the multivectors

$$\mathbf{x} = (x_1,\ldots,x_N) \in \mathbb{R}^{2N}, \quad \mathbf{y} = (y_1,\ldots,y_M) \in \mathbb{R}^{2M} \tag{3}$$

are two clusters of points on the plane z, where $z \in [Z',Z'']$. Thus each point in the set $\{x_n\}$ or $\{y_m\}$ is a two-dimensional transverse position vector. The components of $\mathbf{A} \equiv (\mathbf{a}_0,\mathbf{a}_1,\ldots,\mathbf{a}_L)$ are formal random parameters defined over an appropriate probability space. They describe the randomness and uncertainty existing in our problem. Note that there may be uncertainties that are not due to the turbulence of the medium, but may induce field fluctuations that are difficult to distinguish from the turbulence-related fluctuations. For example, at the plane $z=Z'$, the product (2) becomes

$$\gamma(z=Z',\mathbf{x},\mathbf{y};\mathbf{a}_0) \equiv \prod_{n=1}^{N} \psi(z=Z',x_n;\mathbf{a}_0) \times \prod_{m=1}^{M} \psi^*(z=Z',y_m;\mathbf{a}_0). \tag{4}$$

Here, $\mathbf{a}_0$ denotes the fluctuations due to the source.

Finally we note that in this paper we denote by boldface characters both multivectors and random quantities. Equation (2) is an example, where $\mathbf{x}$ and $\mathbf{y}$ are multivectors and $\mathbf{A}$ is a random parameter that can also be a vector. The reader can determine the type of these quantities from the context.

B. Small- and large-scale averages

We assume that the random parameters can be partitioned into small-and large-scale variabilities: $\mathbf{A} = \{\mathbf{a}_0,\mathbf{A}_l,\mathbf{A}_L\}$. Small-scale variabilities are fluctuations that are smoothed by diffraction and self-averaging due to locally homogeneous fluctuating refraction along a short propagation range. This variability includes intrinsic intermittency effects [4], e.g., fluctuations due to the energy-dissipation rate of the turbulence. Large-scale variability is the fluctuation of the locally homogeneous medium [3]. Thus two types of averages should be considered, the small-scale conditional average for a given frozen large-scale event denoted by $\langle\ |\mathbf{A}_L\rangle_{\mathbf{A}_l}$, and the large-scale average denoted by $\langle\ \rangle_{\mathbf{A}_L}$. The (total) ensemble average is thus written as $\langle\ \rangle \equiv \langle\langle\ |\mathbf{A}_L\rangle_{\mathbf{A}_l}\rangle_{\mathbf{A}_L}$.

C. Moment equations

The governing equation for the evolution of the product of fields propagating in the medium (e.g., Gozani [3]) follows the von Neumann equation:

$$[2ik\partial_{\mathbf{z}} + \mathbf{\Delta_x} - \mathbf{\Delta_y} + k^2 V(z,\mathbf{x},\mathbf{y};\mathbf{A})]\gamma = 0, \tag{5a}$$

with the initial condition

$$\gamma(z=Z',\mathbf{x},\mathbf{y}) = \gamma_0(\mathbf{x},\mathbf{y};\mathbf{a}_0) \tag{5b}$$

defined in (4).

In (5a), $\mathbf{\Delta_s} \equiv \Delta_1 + \cdots + \Delta_d$, where $\mathbf{s}$ is $\mathbf{x}$ or $\mathbf{y}$, and d is N or M, accordingly. The function $V(z,\mathbf{x},\mathbf{y};\mathbf{A})$ describes the scattering potential. It is customary to split it into an average and random parts

$$V(z,\mathbf{x},\mathbf{y};\mathbf{A}) = U(z,\mathbf{x},\mathbf{y}) + \boldsymbol{\nu}(z,\mathbf{x},\mathbf{y};\mathbf{A}), \tag{6a}$$

where

$$U(z,\mathbf{x},\mathbf{y}) \equiv \langle V(z,\mathbf{x},\mathbf{y};\mathbf{A})\rangle, \quad \langle \boldsymbol{\nu}(z,\mathbf{x},\mathbf{y};\mathbf{A})\rangle = 0. \tag{6b}$$

In general, $U(\cdot)$ can be a function of space describing an inhomogeneous medium. By definition, the susceptibility is $\chi \equiv \boldsymbol{\epsilon} - 1$, where $\boldsymbol{\epsilon}$ is the dielectric permittivity. We can write $V(\cdot)$, $U(\cdot)$, and $\boldsymbol{\nu}(\cdot)$ in terms of $\chi(z,x) = \langle \chi(z,x)\rangle + \tilde{\boldsymbol{\epsilon}}(z,x)$, with $\langle \tilde{\boldsymbol{\epsilon}} \rangle = 0$, as follows:

$$\left.\begin{matrix} V \\ U \\ \boldsymbol{\nu} \end{matrix}\right\}(z,\mathbf{x},\mathbf{y}) = \int_{-\infty}^{\infty} d^2x'\,\alpha_\tau(x') \left\{\begin{matrix} \chi(z,x') \\ \langle \chi(z,x')\rangle, \\ \tilde{\boldsymbol{\epsilon}}(z,x') \end{matrix}\right. \tag{7a}$$

where

$$\alpha_\tau(x') = \sum_{n=1}^{N} \delta(x'-x_n) - \sum_{m=1}^{M} \delta(x'-y_m) \tag{7b}$$

is a function of x' and $\mathbf{x}(\tau)$, and $\mathbf{y}(\tau)$.

III. PROPAGATION THROUGH INHOMOGENEOUSLY RANDOM MEDIA

The Green's function for (5a) is the solution for a homogeneous medium to be modulated later by a large-scale process. The Green's function for (5a) and (5b) is written as a Feynman path integral [5]:

$$K(Z'',\mathbf{x}'',\mathbf{y}'';Z',\mathbf{x}',\mathbf{y}')=\int_{Z',\mathbf{x}',y'}^{Z'',\mathbf{x}'',y'}\mathbf{D}^{2N}x(\tau)\mathbf{D}^{2M}y(\tau)$$
$$\times\exp\left\{i\frac{k}{2}\int_{Z'}^{Z''}d\tau[|\dot{\mathbf{x}}(\tau)|^2-|\dot{\mathbf{y}}(\tau)|^2]\right\}$$
$$\times\exp\left\{i\frac{k}{2}\int_{Z'}^{Z''}d\tau\, V[\tau,\mathbf{x}(\tau),\mathbf{y}(\tau);\mathbf{A}]\right\}, \tag{8a}$$

subject to the condition

$$K(z,\mathbf{x}'',\mathbf{y}'';z,\mathbf{x}',\mathbf{y}')=\delta(\mathbf{x}''-\mathbf{x}')\,\delta(\mathbf{y}'-\mathbf{y}''), \tag{8b}$$

and the final solution is obtained by a superposition integral. For example, if only fluctuations of the source are concerned, the result reads

$$\langle\gamma(Z'',\mathbf{x}'',\mathbf{y}'')\rangle=\int d\mathbf{x}'\,d\mathbf{y}'\,K(Z'',\mathbf{x}'',\mathbf{y}'';Z',\mathbf{x}',\mathbf{y}')\langle\gamma_0(\mathbf{x}',\mathbf{y}';\mathbf{a}_0)\rangle_{\mathbf{a}_0}. \tag{8c}$$

In (8a), the first, second, and third factors describe functional integration, diffraction, and refraction, respectively. The present paper is concerned mainly with the third (refractive) factor, thus details of functional integration are unnecessary. Exhaustive details of application of functional integration to propagation through random media are covered elsewhere [6].

The fluctuations of the third term induce fluctuations on $K(\cdot)$. We apply the small-scale average to $K(\cdot)$ and get

$$g(Z'',\mathbf{x}'',\mathbf{y}'';Z',\mathbf{x}',\mathbf{y}')=\langle K(z,\mathbf{x}'',\mathbf{y}'';z,\mathbf{x}',\mathbf{y}')|\mathbf{A}_L\rangle_{\mathbf{A}_l}$$
$$=\int_{Z',\mathbf{x}',\mathbf{y}'}^{Z'',\mathbf{x}'',\mathbf{y}'}\mathbf{D}^{2N}x(\tau)\mathbf{D}^{2M}y(\tau)\exp\left\{i\frac{k}{2}\int_{Z'}^{Z''}d\tau[|\dot{\mathbf{x}}(\tau)|^2-|\dot{\mathbf{y}}(\tau)|^2]\right\}$$
$$\times\exp\left\{i\frac{k}{2}\int_{Z'}^{Z''}d\tau\, U[\tau,\mathbf{x}(\tau),\mathbf{y}(\tau)]\right\}$$
$$\times\left\langle\exp\left\{i\frac{k}{2}\int_{Z'}^{Z''}d\tau\,\boldsymbol{\nu}[\tau,\mathbf{x}(\tau),\mathbf{y}(\tau),\mathbf{A}]\right\}\middle|\mathbf{A}_L\right\rangle_{\mathbf{A}_l}, \tag{9a}$$

where

$$g(z,\mathbf{x}'',\mathbf{y}'';z,\mathbf{x}',\mathbf{y}')=\delta(\mathbf{x}''-\mathbf{x}')\,\delta(\mathbf{y}'-\mathbf{y}''). \tag{9b}$$

The third term in (9a) is the characteristic functional,

$$\Phi_{\boldsymbol{\nu}|\mathbf{A}_L}[\omega(\cdot),\mathbf{x}(\cdot),\mathbf{y}(\cdot);Z',Z'']\equiv\left\langle\exp\left\{i\int_{Z'}^{Z''}d\tau\,\omega(\tau)\boldsymbol{\nu}[\tau,\mathbf{x}(\tau),\mathbf{y}(\tau),\mathbf{A}]\right\}\middle|\mathbf{A}_L\right\rangle_{\mathbf{A}_l}$$
$$=\exp\left\{\sum_{n=1}^{\infty}\frac{i^n}{n!}\int_{Z'}^{Z''}d\tau_1\cdots\int_{Z'}^{Z''}d\tau_n K^{(n)}_{\boldsymbol{\nu}|\mathbf{A}_L}(\tau_1,\mathbf{x}_1,\mathbf{y}_1;\ldots;\tau_n,\mathbf{x}_n,\mathbf{y}_n)\omega_1\cdots\omega_n\right\}, \tag{10a}$$

where

$$K^{(n)}_{\boldsymbol{\nu}|\mathbf{A}_L}(\tau_1,\mathbf{x}_1,\mathbf{y}_1;\ldots;\tau_n,\mathbf{x}_n,\mathbf{y}_n)\equiv\langle\langle\boldsymbol{\nu}(\tau_1,\mathbf{x}_1,\mathbf{y}_1,\mathbf{A})\cdots\boldsymbol{\nu}(\tau_n,\mathbf{x}_n,\mathbf{y}_n,\mathbf{A})|\mathbf{A}_L\rangle\rangle \tag{10b}$$

is the nth-order conditional cumulant (semi-invariant) with respect to $\{\boldsymbol{\nu}[\tau,\mathbf{x}(\tau),\mathbf{y}(\tau);\mathbf{A}]|\mathbf{A}_L\}$. In (10), $\mathbf{x}_n\equiv\mathbf{x}(\tau_n)$, $\mathbf{y}_n\equiv\mathbf{y}(\tau_n)$, and $\omega_n\equiv\omega(\tau_n)=k/2$. In (10b), $\langle\langle\ \rangle\rangle$ denotes cumulant average. Because a longitudinal correlation is kept, the trajectories $\mathbf{x}(\tau)$ and

$\mathbf{y}(\tau)$ are coupled throughout the propagation path. This renders the characteristic functional global, and thus differential equations cannot be obtained. The characteristic functional is written in terms of the characteristic functional of the dielectric permittivity as

$$\Phi_{\boldsymbol{\nu}|\mathbf{A}_L}\left[\omega(\cdot)\equiv\frac{k}{2},\mathbf{x}(\cdot),\mathbf{y}(\cdot);Z',Z''\right]\equiv\Phi_{\tilde{\epsilon}|\mathbf{A}_L}\left[\frac{k}{2}\,\alpha_\tau(\cdot);Z',Z''\right]=\left\langle\exp\left[i\,\frac{k}{2}\int_{Z'}^{Z''}d\tau\int d^2\mathbf{x}'\,\alpha_\tau(\mathbf{x}')\,\tilde{\epsilon}(z,\mathbf{x}')\right]\middle|\mathbf{A}_L\right\rangle_{\mathbf{A}_l}, \quad (11)$$

where $\alpha_\tau(\cdot)$ is defined in (7b); note that $\alpha_\tau(\cdot)$ depends on $\mathbf{x}$ and $\mathbf{y}$ as parameters, and through them on τ. To obtain $\langle\gamma\rangle$, we apply $\langle g(\cdot)\rangle_{\mathbf{A}_L}$, and use it in (8c).

Propagation through a δ-correlated medium

The propagation through a δ-correlated medium is obtained with

$$K^{(n)}_{\boldsymbol{\nu}|\mathbf{A}_L}(\tau_1,\ldots,\tau_n)\simeq K^{(n)}_{\text{eff}}(\tau_1;\mathbf{x}_1,\mathbf{y}_1,\ldots,\mathbf{x}_n,\mathbf{y}_n)\,\delta(\tau_1-\tau_2)\cdots\delta(\tau_{n-1}-\tau_n), \quad (12)$$

where $\mathbf{x}_n\equiv\mathbf{x}(\tau_n)\in\mathbb{R}^{2N}$ and $\mathbf{y}_n\equiv\mathbf{y}(\tau_n)\in\mathbb{R}^{2M}$. This case leads to

$$\Theta_{\text{eff}}\left[\omega(\cdot)\equiv\frac{k}{2},\mathbf{x}(\cdot),\mathbf{y}(\cdot);Z',Z''\right]=\sum_{n=1}^{\infty}\frac{1}{n!}\left(i\,\frac{k}{2}\right)^n\int_{Z'}^{Z''}d\tau\,K^{(n)}_{\text{eff}}[\tau,\mathbf{x}(\tau),\mathbf{y}(\tau)]. \quad (13)$$

A δ-correlated model is an idealization that is pertinent when many longitudinal scales of a homogeneous process are embedded in the correlation length of the field. The latter is of the order of the propagation range, $Z\equiv Z''-Z'$ [7]. The longitudinal correlation of the medium is usually very small compared with the range. Therefore the δ-correlated model applies and the position of $\mathbf{x}(\tau_1)$ and $\mathbf{y}(\tau_2)$ is contracted to the same plane, $\tau=\tau_1=\tau_2$. In this case, we can expand $g(z+\Delta z,\mathbf{x},\mathbf{y};Z',\mathbf{x}',\mathbf{y}')$ in a Taylor series up to $O(\Delta z^2)$ and get

$$\partial_{\mathbf{z}}g(z,\mathbf{x},\mathbf{y};Z',\mathbf{x}',\mathbf{y}')=\frac{i}{2k}\,[\Delta_{\mathbf{x}}-\Delta_{\mathbf{y}}]g(z,\mathbf{x},\mathbf{y};Z',\mathbf{x}',\mathbf{y}')+\left\{\frac{ik}{2}\,U(z,\mathbf{x},\mathbf{y})+\partial_{\mathbf{z}}\Theta_{\text{eff}}\left[\omega\equiv\frac{k}{2},\mathbf{x},\mathbf{y};Z',z\right]\right\}g(z,\mathbf{x},\mathbf{y};Z',\mathbf{x}',\mathbf{y}'), \quad (14a)$$

with the initial condition

$$g(Z',\mathbf{x}'',\mathbf{y}'';Z',\mathbf{x}',\mathbf{y}')=\delta(\mathbf{x}''-\mathbf{x}')\,\delta(\mathbf{y}'-\mathbf{y}''). \quad (14b)$$

A similar partial-differential equation was derived for homogeneously random media by Klyatskin [8]. To obtain $\langle\gamma\rangle$, we apply $\langle g(\cdot)\rangle_{\mathbf{A}_L}$, use it in (8c), and repeat the procedure used to derive (14) to obtain an analogous equation for the large-scale variability.

IV. STATISTICAL MODELS FOR THE SMALL- AND LARGE-SCALE VARIABILITIES

It has been argued in the past [9] and recently confirmed by wavelet decomposition [10] that a turbulent realization can be decomposed orthogonally into a small-scale process that is practically Gaussian, and a large-scale process that accounts for the burstlike coherent structures. Following these findings, we adopt for the small-scale event $\mathbf{A}_l$ a conditional zero-mean homogeneous δ-correlated Gaussian process, $\mathbf{s}(\cdot)$. This assumption is justified because we have many small-scale events in each characteristic scale of $\mathbf{S}(\cdot)$ and consequently, also in the range $Z=Z''-Z'$. For the large-scale event $\mathbf{A}_L$, we introduce a heuristic yet flexible model of the filtered point process $\mathbf{S}(\cdot)$. To account for the cascading of energy from large into small scales, we keep the ensemble and allow $\langle\mathbf{S}(\cdot)\mathbf{s}(\cdot)\rangle\neq 0$.

The large-scale process $\mathbf{S}(\cdot)$ assumes the form

$$\mathbf{S}(R;\{\boldsymbol{\Xi}_n\},\{\mathbf{R}'_n\})=\sum_{n=1}^{\mathbf{N}}W(R,\mathbf{R}_n,\boldsymbol{\Xi}_n),$$
$$\mathbf{R}_n,R\in\mathbb{R}^3,\quad \mathbf{N}\geqslant 1 \quad (15)$$

with $\mathbf{S}(\cdot)=0$ for $\mathbf{N}=0$. It depends on a three-dimensional position vector $R\equiv(z,x)$, where $x\in\mathbb{R}^2$. The number $\mathbf{N}$ follows a point distribution $P\{\mathbf{N}=N\}$. The set of points is distributed with the nonuniform density $f_{\mathbf{R}}(R)$. The set $\{W(\cdot)\}$ consists of functions with compact support, centered around the random points $\{R_j\}$, $j=1,...,\mathbf{N}$. It is chosen according to the prevalent feature observed visually or with a pattern recognition process, and finally shaped in accordance to spectral demands, similarity laws, etc. The parameter set $\{\boldsymbol{\Xi}_j\}$ consists of independent, identically distributed random vectors that modulate all the properties of $W(\cdot)$. The components of Ξ can describe local features such as local frequency, amplitude, duration, slopes, contour moments: centroid, moment of inertia, etc., and can also switch between sets of functions. The distribution of the parameters $f_{\Xi}(\Xi)$, as well as the local distribution of the bursts $f_{\mathbf{R}}(R)$, must be found and parametrized from temperature data. Finally, we also allow $f_{\mathbf{R}}(R)$ to be random by denoting it by $\mathbf{f}_{\mathbf{R}}(R)$. We assume that our propagation process evolves in an effective scattering volume $V_{\mathbf{s}}$ that is smaller than the volume V on which $\mathbf{f}_{\mathbf{R}}(R)$ is defined, i.e., $\int_{\mathbf{V}} d^3R\ f_{\mathbf{R}}(R)=1$. Roughly speaking, the shape of $V_{\mathbf{s}}$ is a narrow tube with a radius of the first Fresnel zone.

The filtered point process is convenient for propagation studies because it can be used to synthesize a quite arbitrary, burstlike phenomenon, and it has a characteristic functional in closed analytical form. Using a sequence of conditional averages in the order $\{\Xi_n\}$, $\{\mathbf{R}_n\}$, $\mathbf{N}$, and $\mathbf{f}_{\mathbf{R}}$, it can be shown that the conditional characteristic functional for given $\mathbf{f}_{\mathbf{R}}=f_{\mathbf{R}}$ is

$$\Phi_{\tilde{\epsilon}|\mathbf{f}_{\mathbf{R}}}\left[\frac{k}{2}\ \alpha_\tau(\cdot);Z',Z''\right]=\langle \mathbf{a}[f_{\mathbf{R}'}]^{\mathbf{N}}\rangle_{\mathbf{N}}, \tag{16a}$$

$$\mathbf{a}[f_{\mathbf{R}'}]\equiv 1+\int_{V_{\mathbf{s}}} d^3R' f_{\mathbf{R}'}(R')\left\{\Phi_{\Xi,\mathbf{s}}\left[\frac{k}{2}\ \alpha_\tau(\cdot),R,\mathbf{R}'\right]-1\right\}, \tag{16b}$$

$$\Phi_{\Xi,\mathbf{s}}\left[\frac{k}{2}\ \alpha(\cdot),R,\mathbf{R}'\right]\equiv\left\langle \Phi_{\mathbf{s}|\Xi,\mathbf{R}'}[\alpha_\tau(\cdot),Z',Z'']\ \exp\left[i\ \frac{k}{2}\int_{Z'}^{Z''}d\tau\int_{x\in V_{\mathbf{s}}}d^2x\ \alpha_\tau(x)W(\tau,x,\mathbf{R}',\Xi)\right]\right\rangle_{\Xi}. \tag{16c}$$

In (16b), $V_{\mathbf{s}}$ is the effective scattering volume, and in (16c)

$$\Phi_{\mathbf{s}|\Xi,\mathbf{R}'}\left[\frac{k}{2}\ \alpha_\tau(\cdot);Z',Z''\right]\equiv\left\langle \exp\left[i\ \frac{k}{2}\int_{Z'}^{Z''}d\tau\int_{x'\in V_{\mathbf{s}}}d^2x\ \alpha_\tau(x)\mathbf{s}(\tau,x,\mathbf{R}',\Xi)\right]\middle|\Xi\right\rangle_{\mathbf{s}} \tag{17a}$$

$$=\exp\left\{-\frac{k^2}{8}\int_{Z'}^{Z''}d\tau\int_{x_1,x_2\in V_{\mathbf{s}}}d^2x_1d^2x_2\alpha_\tau(x_1)\alpha_\tau(x_2)\mathbf{A}_{\mathbf{s}|\Xi,\mathbf{R}'}[x_1(\tau)-x_2(\tau)]\right\}, \tag{17b}$$

where the assumption of a homogeneous δ-correlated $\mathbf{s}(\cdot)$ leads to

$$\mathbf{A}_{\mathbf{s}|\Xi,\mathbf{R}'}[x_1(z)-x_2(z)]\simeq\int_{-\infty}^{\infty}d\zeta\langle\mathbf{s}[z,x_1(z);\mathbf{R}',\Xi]\times\mathbf{s}[z+\zeta,x_2(z+\zeta);\mathbf{R}',\Xi]|\Xi\rangle_{\mathbf{s}}. \tag{17c}$$

Examples

Consider now a few examples that seem to be appropriate for large-scale events of refractivity. We take a large volume V over which reliable statistics of the point process can be defined. Then we define a probability of occurrence of an elementary event in V to be $p\equiv V_{\mathbf{e}}/V$, i.e., $0<p<1$, and define $q\equiv 1-p$. The case of m events in V out of N independent events in V is described by the binomial variate $\mathbf{m}$, with $P\{\mathbf{m}=m|N\}=\binom{N}{m}p^mq^{N-m}$, $0\leqslant m\leqslant N$. In this case, N is fixed and we are looking for $\langle\mathbf{a}^{\mathbf{m}}\rangle_{\mathbf{m}}$. Thus the characteristic functional reads

$$\Phi_{\tilde{\epsilon}|\mathbf{f}_{\mathbf{R}}}\left[\frac{k}{2}\ \alpha_\tau(\cdot);Z',Z''\right]=\sum_{m=0}^{N}\binom{N}{m}p^mq^{N-m}\mathbf{a}^m=(p\mathbf{a}+q)^N, \tag{18}$$

where $\mathbf{a}$ is defined in (16) and (17). Now we apply $\langle\Phi_{\tilde{\epsilon}|\mathbf{f}_{\mathbf{R}}}[\cdot]\rangle_{\mathbf{f}_{\mathbf{R}}}$, and finally the resulting $\Phi_{\tilde{\epsilon}}[\cdot]$ is inserted into the expression (9a) of the path integral.

An important example is the generalized Poisson process for inhomogeneous refractivity. This model can include multiple scattering by aerosols [11] in addition to the turbulence and the large-scale structures. The binomial variate tends to the Poisson variate as $N\to\infty$ and $p\equiv V_{\mathbf{e}}/V\ll 1$, for $m=O(Np)$ [12]. The limiting distribution is $P\{\mathbf{m}=m\}=e^{-\langle\mathbf{m}\rangle}\langle\mathbf{m}\rangle^m/m!$. For a nonuniform distribution of bursts, with $N\to\infty$, $V\to\infty$, and $\mathbf{p}\equiv\int_{\mathbf{V_e}}dR\ \mathbf{f}_{\mathbf{R}}(R)\ll 1$, we define the intensity $\mathbf{\Lambda}(R)\equiv\langle\mathbf{N}\rangle\mathbf{f}_{\mathbf{R}}(R)$ and obtain the characteristic functional of the generalized Poisson process,

$$\Phi_{\tilde{\epsilon}}\left[\frac{k}{2}\ \alpha_\tau(\cdot);Z',Z''\right]=\left\langle\exp\left(\int_{V_s}d^3R\ \mathbf{\Lambda}(R)\left\{\Phi_{\Xi,\mathbf{s}}\times\left[\frac{k}{2}\ \alpha_\tau(\cdot);Z',Z'',R\right]-1\right\}\right)\right\rangle_{\mathbf{\Lambda}}. \tag{19}$$

Many discrete distributions can be used to generalize (16) and (18). Any function $u(t)$, $0<t<1$, that admits Maclaurin series with positive coefficients can be used to create a characteristic functional $u(t\mathbf{a})/u(t)$ [12]. In particular, by using the binomial variate it is possible to obtain the characteristic functional for $N=0,1,...$ large-scale events along the path. It is anticipated that a small N, e.g., 1–4, will produce a significant effect, whereas a large N, e.g., $N>10$, will homogenize the path. In this case the large-scale δ-correlated model is justified, and a large-scale partial-differential equation can be obtained using the procedure mentioned in (14). The multiple large-scale δ-correlated version of the filtered point process is obtained by inserting $W(z,x,\mathbf{R}',\Xi)=W(z,x,\mathbf{x}',\Xi)\delta(z-\mathbf{z}')$ in (15), and carrying out the analysis analogous to (12)–(14).

V. CONCLUSIONS

The theory presented here provides closed form expressions that are useful in parametrizing turbulent temperature data for realistic propagation scenarios, and for validating simulations of propagation through inhomogeneously random media. The correspondence of these expressions with a readily generated, albeit flexible model, can contribute to a controlled study of this phenomenon. Although the formulation is rather complicated, it appears that it can be computed for $N=M=1$, and $N=M=2$ in (2)–(4), which are of a great interest in imaging, energy channeling, and remote sensing studies.

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